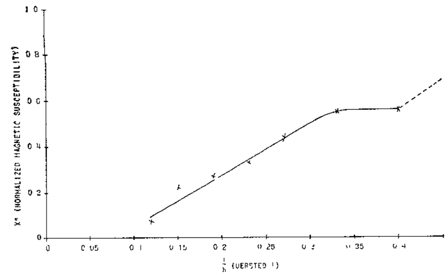

Fig. 5—Dependence of magnetic susceptibility on the reciprocal of the RF magnetic field at high power levels for type-YIG material.

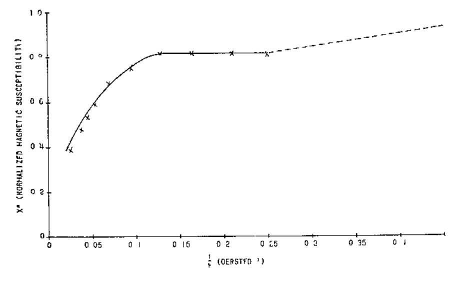

Fig. 6—Dependence of the magnetic susceptibility on the reciprocal of the RF field at high power levels for type R1 material.

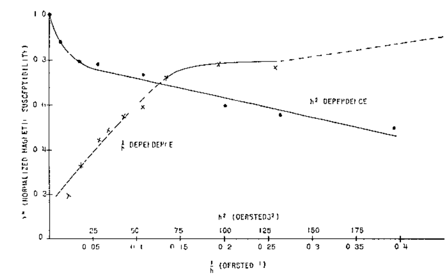

Fig. 7—Dependence of the magnetic susceptibility on the square of the RF field at low power levels, and on the reciprocal of the RF field at high power levels for type-414 material.

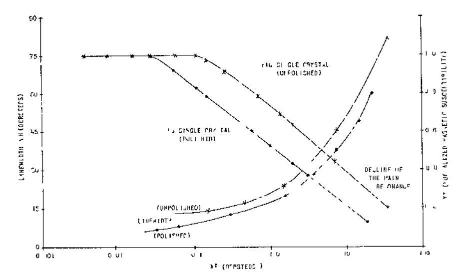

Fig. 8—Observed linewidth and decline of the main resonance as a function of incident power (square of RF field) for single crystal type-YIG material.

material is naturally reflected in its figure of merit [Table II, 8)] which, rather than decreasing in the expected manner, increases with incident power.

Type 447 material has been successfully used in high-power device applications. Since the linewidth and resonance absorption remain fairly constant with incident power, one would expect this material to provide satisfactory device characteristics at high power levels.

TABLE III

CRITICAL RF FIELDS ($h_c$) AND CORRESPONDING SPIN-WAVE RESONANCE LINEWIDTH ($\Delta H_k$)

| Material | $4\pi M_s$ Oersteds | $\Delta H$ Oersteds | $h_c$ Oersteds | $\Delta H_k$ Oersteds |
|---|---|---|---|---|
| 414 | 650 | 166 | 5.2 | 0.13 |
| 551–16 | 1100 | 143 | 4.9 | 0.16 |
| YIG | 1800 | 44 | 1.6 | 0.3 |
| R1 | 2100 | 505 | 9.2 | 0.09 |
| Y1–8 | 1800 | 56 | 1.8 | 0.23 |
| Single-Crystal YIG, Unpolished | 1750 | 13.5 | 0.33 | 0.13 |
| Single-Crystal YIG, Polished | 1750 | 6.5 | 0.17 | 0.16 |

$4\pi M_s$ = saturation magnetization
$\Delta H$ = linewidth of sample
$h_c$ = RF field
$\Delta H_k$ = spin-wave resonance linewidth

The curves in Figs. 2 and 4 show an erratic behavior at "medium" field and power levels, which is not caused by experimental deviations. The fine structure noted in this region should be investigated further in order to determine the reason for the anomalous behavior.

Green and Schlömann[6] have shown that at fairly low power levels the susceptibility varies linearly with the square of the RF magnetic field strength, and that at high power levels, the susceptibility is inversely proportional to the amplitude of the RF magnetic field strength. The representative data presented in Figs. 5 and 6 tend, in general, to confirm these results.

The dependence of the susceptibility on $1/h$ at high power levels was compared to the linear relationship with the $h^2$ dependence at low power levels. The results for a representative sample are shown in Fig. 7. In all cases, the values for the magnetic susceptibility $X''$ are normalized to the low-power value.

The data presented can also be used to determine the critical field strength and the corresponding spin-wave resonance linewidth by use of the method of Schlömann, Saunders, and Servetz.[7] These values for the materials investigated are presented in Table III. It is interesting to note that the variations in $\Delta H_k$ are much smaller than those in $\Delta H$.

The only single-crystal material investigated was yttrium-iron garnet One sample of this material was checked both before and after it was polished. The results, which are given in Fig. 8, show the dependence of the linewidth and the decline of the main resonance on incident RF power. The results also show the expected effect of surface finish for narrow linewidth materials. The effectiveness of a polished surface in reducing linewidth is evident.

## CONCLUSIONS

Information has been obtained concerning the dependence of linewidth and the decline of the main resonance on incident microwave power level for several common types of ferromagnetic materials.

The anomalous behavior noted in these experiments indicates the need for additional theoretical and experimental study. An indication has also been obtained of the direction to be taken in ferromagnetic material research in order to achieve optimized performance of microwave ferromagnetic devices, such as isolators, circulators, power limiters, and parametric amplifiers.

J. L. CARTER
S. DIXON, JR.
I. REINGOLD
U. S. Army Signal Res. and Dev. Lab.
Fort Monmouth, N. J.

[6] J. J. Green and E. Schlomann, "High Power Ferromagnetic Resonance at X-Band in Polycrystalline Garnets and Ferrites," Raytheon Co., Waltham, Mass., Tech. Memo. T-168; 1959.

[7] E. Schlomann, J. Saunders, and M. Servetz, "L-Band Ferromagnetic Resonance Experiments at High Peak Power Levels," Raytheon Co., Waltham, Mass., Tech. Memo. T-167; 1959.

## Green's Function Techniques for Inhomogeneous Anisotropic Media*

In many problems involving the guiding and radiation of electromagnetic waves the solution for the field quantities at points in space is given in terms of integrals of the field quantities over their values on a closed surface. These integrals are often derived through the application of vector Green's theorems. The Green's function used in any particular application is usually determined by the special considerations of that problem, but it is convenient to use, as the Green's function, a solution of the vector wave equation which is singular at the point where the field is to be computed. In this article the concept is extended to include media which are anisotropic and may be inhomogeneous as well. Use is made of the generalized reciprocity relationships for anisotropic media.[1] This involves the use of the media of a given problem termed "original media" and those characterized by transposed tensor parameters and termed "transposed media."

If the media for a given problem are anisotropic with tensor constitutive parameters which are not necessarily symmetric, the following identity forms a convenient starting point:

* Received by the PGMTT, August 8, 1960; revised manuscript received November 28, 1960. This work is condensed from a chapter of a dissertation submitted in partial fulfillment of the requirements for the Ph.D. degree, Syracuse University, Syracuse, N. Y. It was performed at Syracuse University and was sponsored by the Office of Ordnance Res., U. S. Army, Contract No. DA-30-115-ORD-861.

[1] R. F. Harrington and A. T. Villeneuve, "Reciprocity relationships for gyrotropic media," IRE TRANS. ON MICROWAVE THEORY AND TECHNIQUES, vol. MTT-6, pp 308–310; July, 1958.

$$\iiint_V (\overline{B}\cdot\nabla\times[\phi]\nabla\times\overline{A} - \overline{A}\cdot\nabla\times[\phi]\nabla\times\overline{B})dv = \oiint_s (\overline{A}\times[\phi]\,\nabla\times\overline{B} - \overline{B}\times[\tilde{\phi}]\nabla\times\overline{A})\cdot d\overline{S}, \quad (1)$$

where $\overline{A}$ and $\overline{B}$ are vector functions of position, and the surface integral extends over the surface enclosing the volume $V$. Here $[\phi]$ is a tensor function of position and is not necessarily symmetric. The tilde indicates the transposed tensor. This identity may be derived by applying the divergence theorem to

$$\iiint_V \nabla\cdot(\overline{A}\times[\phi]\nabla\times\overline{B} - \overline{B}\times[\tilde{\phi}]\nabla\times\overline{A})dv, \quad (2)$$

and transforming the volume integral by the identity

$$\nabla\cdot(\overline{F}\times\overline{H}) = \overline{H}\cdot\nabla\times\overline{F} - \overline{F}\cdot\nabla\times\overline{H}. \quad (3)$$

Now let $\overline{B}\equiv\overline{E}$ and $\overline{A}\equiv\overline{G}$, where $\overline{E}$ is the desired electric field intensity due to sources outside of $V$, and $\overline{G}$ is arbitrary. If $[\phi] \equiv [\mu]^{-1}/j\omega$, (1) becomes

$$\iiint_V (\overline{E}\cdot\nabla\times[\tilde{\mu}]^{-1}\nabla\times\overline{G} - \overline{G}\cdot\nabla\times[\mu]^{-1}\nabla\times\overline{E})dv = \oiint_s (\overline{G}\times[\mu]^{-1}\nabla\times\overline{E} - \overline{E}\times[\tilde{\mu}]^{-1}\nabla\times\overline{G})\cdot d\overline{S}. \quad (4)$$

Now let $\overline{G}$ be a solution of the equation

$$\nabla\times[\tilde{\mu}]^{-1}\nabla\times\overline{G} - \omega^2[\tilde{\epsilon}]\overline{G} = -\,\overline{u}_p\delta(|\overline{r}-\overline{r}'|), \quad (5)$$

where $\delta(|\overline{r}|)$ is the Dirac delta function and $\overline{r}'$ is within $V$. The tensors $[\mu]$ and $[\epsilon]$ are, respectively, the tensor permeability and permittivity of the original media in which the fields must be determined, $\overline{u}_p$ is a constant unit vector, and $\omega$ is the angular frequency. Time variations of the form $e^{j\omega t}$ are assumed. Since $\overline{G}$ is singular at $\overline{r}=\overline{r}'$, this point must be excluded from $V$ by enclosing it within a small sphere of radius $\sigma$ as in Fig. 1. In the remaining volume $V_1$, the volume integrand vanishes identically and the identity of (4) becomes

$$\iint_\sigma (\overline{G}\times[\mu]^{-1}\nabla\times\overline{E} - \overline{E}\times[\tilde{\mu}]^{-1}\nabla\times\overline{G})\cdot d\overline{S} = -\iint_s (\overline{G}\times[\mu]^{-1}\nabla\times\overline{E} - \overline{E}\times[\tilde{\mu}]^{-1}\nabla\times\overline{G})\cdot d\overline{S}. \quad (6)$$

On taking the limit of the left-hand side as $\sigma\rightarrow 0$, and in view of the singularity assumed in (5), one arrives at the result

$$\overline{u}_p\cdot\overline{E}(\overline{r}') = -\oiint_s (\overline{G}\times[\mu]^{-1}\nabla\times\overline{E} - \overline{E}\times[\tilde{\mu}]^{-1}\nabla\times\overline{G})\cdot d\overline{S}, \quad (7)$$

which is the component of $\overline{E}(\overline{r}')$ along the unit vector $\overline{u}_p$. Since $\overline{u}_p$ may be arbitrarily oriented, this amounts to a complete determination of $\overline{E}(\overline{r}')$. This expression is analogous to expressions occurring in the isotropic case with two important differences. First, $[\mu]$ is a tensor, and second, $\overline{G}$ now satisfies a vector wave equation in the transposed media rather than in the original media.

The foregoing was applied to a closed source-free region. It was found that the total field could be expressed in terms of the Green's function and of the fields on the surface bounding the region. Next consider a region with a source as shown in Fig. 2. The technique will be formulated for a point in $V'$, the volume bounded by $S$ and $\Sigma$. Call the volume bounded by $\sigma$, $S$ and $\Sigma$, $V_2$, and let $\overline{J}^i$ be an impressed electric current density within $V_2$. Therefore in $V'$ the electric field satisfies

$$\nabla\times[\mu_2]^{-1}\nabla\times\overline{E} - \omega^2[\epsilon_2]\overline{E} = -j\omega\overline{J}^i. \quad (8)$$

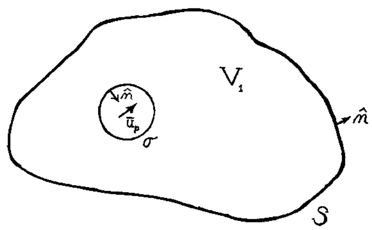


Fig. 1.

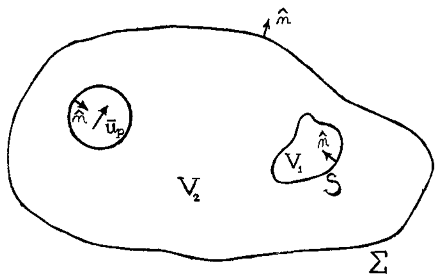


Fig. 2.

With these definitions a procedure similar to that above yields

$$\overline{u}_p\cdot\overline{E}(\overline{r}') = \iiint \overline{G}_2\cdot\overline{J}^i dv - \iint_{\Sigma+s} (\overline{G}_2\times[\mu_2]^{-1}\nabla\times\overline{E} - \overline{E}\times[\tilde{\mu}_2]^{-1}\nabla\times\overline{G}_2)\cdot d\overline{S}. \quad (9)$$

Thus, the field at $\overline{r}'$ in $V'$ is completely determined. If $\Sigma$ recedes to infinity it may be shown that the integral over this surface vanishes[2] with the result

$$\overline{u}_p\cdot\overline{E}(\overline{r}') = \iiint \overline{G}_2\cdot\overline{J}^i dv - \iint_s (\overline{G}_2\times[\mu_2]^{-1}\nabla\times\overline{E} - \overline{E}\times[\tilde{\mu}_2]^{-1}\nabla\times\overline{G}_2)\cdot d\overline{S}. \quad (10)$$

[2] J. R. Mentzer, "Scattering and Diffraction of Radio Waves," in Pergamon Sci Ser., "Electronics and Waves," Pergamon Press, Inc., New York, N. Y., vol. 7, pp. 12–22; 1955.

Through the modified reciprocity theorem the first term on the right-hand side of (10) becomes

$$\iiint \overline{G}\cdot\overline{J}^i dv = \iiint \overline{u}_p\cdot\overline{E}^i(\overline{r})\delta(|\overline{r}-\overline{r}'|)dv = \overline{u}_p\cdot\overline{E}^i(\overline{r}'). \quad (11)$$

Thus, (10) becomes

$$\overline{u}_p\cdot\overline{E}(\overline{r}') = \overline{u}_p\cdot\overline{E}^i(\overline{r}') - \oiint_s (\overline{G}_2\times[\mu_2]^{-1}\nabla\times\overline{E} - \overline{E}\times[\tilde{\mu}_2]^{-1}\nabla\times\overline{G}_2)\cdot d\overline{S}. \quad (12)$$

This is an example of a modified Green's function technique which may be applied to anisotropic media. The only difference between this technique and that for isotropic media is that the Green's functions used satisfy equations for the transposed media.

Alfred T. Villeneuve
Hughes Aircraft Co.
Culver City, Calif.

## $N$-Way Power Divider*

An $N$-way power divider is an $(N+1)$ port network with $N$ equal outputs. If we assume the $N$ outputs are symmetrical and the input port (1) is matched, then

$$S = \begin{bmatrix} 0 & S_{12} & S_{12} & S_{12}\cdots S_{12} \\ S_{12} & S_{22} & S_{23} & S_{23}\cdots S_{23} \\ S_{12} & S_{23} & S_{22} & \cdots\cdots S_{23} \\ S_{12} & S_{23} & S_{23} & \\ \cdots & \cdots & \cdots & \cdots S_{23} \\ S_{12} & S_{23} & S_{23} & S_{23}\cdots S_{22} \end{bmatrix}$$

which is a scattering matrix of the $(N+1)$th order. Since the device is lossless, the matrix is unitary. Therefore,

$$N|S_{12}|^2 = 1$$

$$|S_{12}|^2 + |S_{22}|^2 + (N-1)|S_{23}|^2 = 1$$

$$S_{11}S_{12}^* + S_{12}S_{22}^* + (N-1)S_{12}S_{23}^* = 0.$$

The solution to this simultaneous equation indicates that

$$|S_{12}|^2 = \frac{1}{N}$$

$$|S_{23}|^2 = \frac{1}{N^2}$$

$$|S_{22}| = \frac{N-1}{N}.$$

* Received by the PGMTT, December 13, 1960.